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Minkowski spacetime does not apply to a homogeneously accelerating medium

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abstract

Home and comoving inertial frame parameters of an individual point of an idealized medium of launch length L uniformly co-accelerating between identical fixed-thrust rockets, are well known. This is not the case with the varying inter-rocket radar periods and related implications regarding a changing 'non-inertial own-length' K which differs from a front rocket's retrospective separation L from the simultaneously relatively moving rear rocket. On the other hand, the nonhomogeneous acceleration case involving every comoving frame's unchanging perception of a contrived 'rigor mortis' medium (so-called 'rigid motion' traditionally associated with 'Rindler coordinates') whereby K ¼ L ¼ L, constitutes the sole extended accelerating medium scenario where the entrenched Minkowski metric is actually applicable. Paraphrasing Wolfgang Pauli, not only is Minkowski spacetime not correct [in the general sense], it is not even wrong [in the restricted sense].

Introduction

Ever since Max Born [1] in a 56 page treatise dedicated to the memory of Hermann Minkowski adopted the famous 'Minkowski metric' in the context of the motion of a 'rigid electron', a point—not an extended—object, Minkowski spacetime has been indiscriminately allocated a dominant role especially in the literature of general relativity. A 2006 book Special Relativity - Will it Survive the Next 101 Years? ([14], p. 40), claimed that "...the geometry of Minkowski spacetime has been axiomatized in all rigour", yet a 2000 book by the same publisher—The Geometry of Spacetime by Callahan ([8], p. 196)—however states: "Spacetime with gravity [i.e. general relativity] does not obey the laws of Minkowski geometry...".

The present paper argues in an entirely orthodox way that, notwithstanding the metric's vindicated role in particle physics, Minkowski's landmark equation is overgeneralised even in flat spacetime, a negligence which has led to the unwarranted yet widespread opinion that "Special relativity is not equipped to describe observations in noninertial frames" (Sartori's 1996 Understanding Relativity [7], pp. 183–4). It has also prevented—and continues to prevent—proper progress in the domain of relativistic acceleration of extended media.

'Rigor mortis' acceleration

In his classic 2001/2006 textbook Relativity, Special, General and Cosmological [13] (Section 3.8: 'Rigid motion and the uniformly accelerated rod'), Rindler deploys the Minkowski metric to an extended medium ([13]'s Eqs. (2.13)/(2.14)). Its constituent parts are assumed to accelerate at fixed own-acceleration rates[1] which individually differ so that any comoving inertial frame set of observers would measure the medium's 'increments' as all momentarily stationary, and also view the medium as having maintained its original 'rest length' at launch. The required relationship between the fixed own-acceleration of each increment of the medium and its rearmost end—the scaled separation being equal to the (likewise scaled) difference of the two acceleration inverses—did not appear in [13]. The relationship was however explicitly presented in a four-velocities context earlier in 2002 by Woodhouse ([5], p. 115). Moreover in 2010 it was derived by Franklin ([18], Eq. (22)) using equations equivalent to (1)–(4) below.

In the present paper this relationship is established in a comparably direct manner on the physical basis of radar periods rather than purely mathematically, taking into consideration that quantitative definitions of length and time are based on electromagnetic waves (where signal source and observer are of course assumed to

[1] Using prefix own- (German eigen-) for 'proper', a term carelessly adopted from the ambivalent French 'propre'—'proprietary' or 'correct'—and in English relativistically dysfunctional.

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be nonaccelerating). We shall refer to this kernel equation as THE DIFFERENTIAL INVERSE OWN-ACCELERATIONS CONDITION. Avoiding confusions associated with the expression 'rigid motion', and to underline its 'one off' character as opposed to its assumed paramount role in relativity, we designate this motion setup as 'RIGOR MORTIS' ACCELERATION, thus signifying not alone 'stiffness' but also *shared simultaneity in comoving frames*. In spite of being undoubtedly a useful paradigm by virtue of the medium's composite 'observability' in comoving *inertial* frames, this *nonhomogeneous* acceleration scenario could arguably be viewed as 'contrived' so as *to suit the eye of the beholder(s)*. In this paper we contrast it with the strictly *homogeneous* acceleration case—indisputably significant as the 'Occam's razor' paradigm scenario, yet strangely not consistently dealt with in relativity literature.

Relativistic 'contraction' and 'expansion' confusions

Recently elaborated upon by Redžić in [22] and by Franklin in [18], the 'Lorentz/FitzGerald contraction' concept had been firmly remonstrated against by Einstein already in 1911: *"Recently V. Varičak published remarks in [Physikalische Zeitschrift] which may not go unanswered since they could cause confusion. ...The question as to whether the Lorentz contraction really exists or not is misleading. It indeed is not 'real' in that it does not exist for a comoving observer; it is however 'real' in that it could in principle be confirmed by physical means, for a non-comoving [i.e. relatively moving] observer."* [2]. A simple rhetorical question illustrates the futility of the relativistic contraction idea taken literally: *Which of several differently moving observers decides to what degree an observed passing rod 'contracts' ?*

Yet [18] (p. 294) held that a homogeneously accelerating medium's expanding length is '$\gamma d = d_{max}$'. This misconception, which has its origins in *misappropriation of the present tense* (⇒ [17]), continues to be widely endorsed right across the spectrum of relativity literature, a renowned current textbook example being [13].[2] However conclusions below directly drawn from the radar formulae (⇒ Remarks in Section "Photon crossing rates"), show unequivocally that *during acceleration* the uniformly accelerating medium's physical length expansion *cannot* be the γ factor. Although already outlined by the present author in seminar presentations [17,20]—which have encountered perplexing intransigencies, the still 'open' matter of an accelerating medium's expanding length (better known as 'Bell's string paradox') will be treated in a separate work [24].

The present paper's 'radar approach' to the two above mentioned acceleration scenarios, as well as leading to the differential accelerations condition, establishes *radar formulae* (11), (12), (22) and (23) which, like the shared *velocity loci* Eqs. (19)i and (19)ii included in Fig. 1's home frame spacetime chart, the *shutdown frame 'gap' distance* (28), *retrospective rocket distance* (29) and *a homogeneously accelerating medium's expansion conditions* (34), are to the best of the author's knowledge not apparent in the literature.

Spacetime parameters of a fixed thrust rocket

First we recall the derivation of the well known relationships between the parameters for a fixed acceleration point object. A fixed thrust rocket launch frame's velocity v, home time t and home distance x travelled under acceleration and its own-time τ and its *retrospectively perceived* 'retrodistance' χ *from its launch position* $x_0 = 0$ (⇒ Section "The symmetrical spacetime chart viewpoint") as measurable simultaneously in each own comoving inertial frame, are all *zero* at launch. Scaling time so that limit speed c is *one* and with α as the rocket's constant own-acceleration, from *relativistic velocity composition* as $\Delta\tau \to 0$: $v + \Delta v \approx \frac{v+\alpha\cdot\Delta\tau}{1+v\cdot\alpha\cdot\Delta\tau}$ and hence $dv/d(\tau\alpha) = 1 - v^2$. Accordingly $v = \tanh\tau\alpha$, $\gamma = 1/\sqrt{1-v^2} = \cosh\tau\alpha$ and $v\gamma = \sinh\tau\alpha$.

Using the inverse Lorentz transformation $t = (\tau + \chi v)\gamma$, for a *point* object ($\Delta\chi = 0$), $\Delta\tau\alpha = \Delta t\alpha/\gamma$, yields $t\alpha = \sinh\tau\alpha$. Since $\tanh\tau\alpha = v = \frac{dx}{dt} = \frac{dx}{d\tau}/\frac{dt}{d\tau} = \frac{dx}{d\tau}/\gamma = \frac{dx}{d\tau}/\cosh\tau\alpha$, then $\frac{d(x\alpha)}{d(\tau\alpha)} = \sinh\tau\alpha$ and $x\alpha = \int_0^{\tau\alpha}\sinh\tau\alpha \cdot d(\tau\alpha) = \cosh\tau\alpha - 1$. The familiar equations are therefore:

$$t\alpha = \sinh\tau\alpha = v\gamma, \tag{1}$$

$$v = \tanh\tau\alpha = t\alpha/\sqrt{1+(t\alpha)^2}, \tag{2}$$

$$\gamma = \cosh\tau\alpha = \sqrt{1+\sinh^2\tau\alpha}, \tag{3}$$

$$x\alpha = \cosh\tau\alpha - 1 = \gamma - 1 \qquad \text{and} \tag{4}$$

$$(x\alpha+1)^2 - (t\alpha)^2 = 1. \tag{5}$$

Inter-rocket photon trajectories

We now consider two rockets accelerating *differently*. Denoting the rear and front rocket arbitrary fixed own-accelerations as α_r and α_f, we use variable ρ to assign to a light photon[3] (itself of course *timeless*) various rocket own-times: on emission ($\acute{\rho}$), reflection ($\hat{\rho}$), return/re-emission ($\grave{\rho}$) and re-reflection ($\check{\rho}$). From Eqs. (1) and (4), as a photon is emitted from the rear rocket r at any arbitrary own-time $\acute{\rho} = \frac{\sinh^{-1}\acute{t}_r\alpha_r}{\alpha_r}$, i.e. at home-frame time $\acute{t}_r = \frac{\sinh\acute{\rho}\alpha_r}{\alpha_r}$, r will have travelled under acceleration to a home-frame position $\acute{x}_r = \frac{\cosh\acute{\rho}\alpha_r - 1}{\alpha_r}$ since $t_0 (= 0)$. The same photon arrives at the front rocket f at own-time $\hat{\rho} = \frac{\sinh^{-1}\hat{t}_f\alpha_f}{\alpha_f}$, i.e. at home-frame time $\hat{t}_f = \frac{\sinh\hat{\rho}\alpha_f}{\alpha_f}$ when f will have home-frame position $\hat{x}_f + L = \frac{\cosh\hat{\rho}\alpha_f - 1}{\alpha_f} + L$. Since the photon travels at unit limit speed *in the inertial home frame*, then $(\hat{x}_f + L - \acute{x}_r) = (\hat{t}_f - \acute{t}_r)$ i.e.

$$\frac{\cosh\hat{\rho}\alpha_f - 1}{\alpha_f} + L - \frac{\cosh\acute{\rho}\alpha_r - 1}{\alpha_r} = \frac{\sinh\hat{\rho}\alpha_f}{\alpha_f} - \frac{\sinh\acute{\rho}\alpha_r}{\alpha_r}.$$

$$\text{Hence} \qquad \frac{e^{-\hat{\rho}\alpha_f}}{\alpha_f} = \frac{e^{-\acute{\rho}\alpha_r}}{\alpha_r} + \frac{1}{\alpha_f} - \frac{1}{\alpha_r} - L. \tag{6}$$

The reflected photon meets rear rocket r at home time $\grave{t}_r = \frac{\sinh\grave{\rho}_r\alpha_r}{\alpha_r}$ and home frame *position* $\grave{x}_r = \frac{\cosh\grave{\rho}_r\alpha_r - 1}{\alpha_r}$, over equal home-frame distance and time intervals $(\hat{x}_f + L) - \grave{x}_r = \grave{t}_r - \hat{t}_f$:

$$\frac{\cosh\hat{\rho}\alpha_f - 1}{\alpha_f} + L - \frac{\cosh\grave{\rho}_r\alpha_r - 1}{\alpha_r} = \frac{\sinh\grave{\rho}_r\alpha_r}{\alpha_r} - \frac{\sinh\hat{\rho}\alpha_f}{\alpha_f}.$$

Therefore

$$\frac{e^{\hat{\rho}\alpha_f}}{\alpha_f} = \frac{e^{\grave{\rho}\alpha_r}}{\alpha_r} + \frac{1}{\alpha_f} - \frac{1}{\alpha_r} - L \qquad \text{i.e.} \qquad \frac{e^{\hat{\rho}\alpha_f}}{\alpha_f} - \frac{1}{\alpha_f} + \frac{1}{\alpha_r} + L = \frac{e^{\grave{\rho}\alpha_r}}{\alpha_r}. \tag{7}$$

Imagining this photon to itself be reflected again i.e *re-emitted* forward toward the front rocket, by replacing $\acute{\rho}$ with $\grave{\rho}$ and $\hat{\rho}$ with $\check{\rho}$ respectively in (6) we obtain for the front rocket *re-reflection* time $\check{\rho}$:

[2] Rindler in 2006 ([13] p.76, exercise 3.24) implied that the γ factor expansion is applicable—without referring to time dispersal or rocket motor shutdowns i.e. during acceleration (⇒ Section " The front rocket's 'retrospective separation' " below).

[3] Radar signals use wave pulses rather than photons—a descriptively useful 'equivalent'.

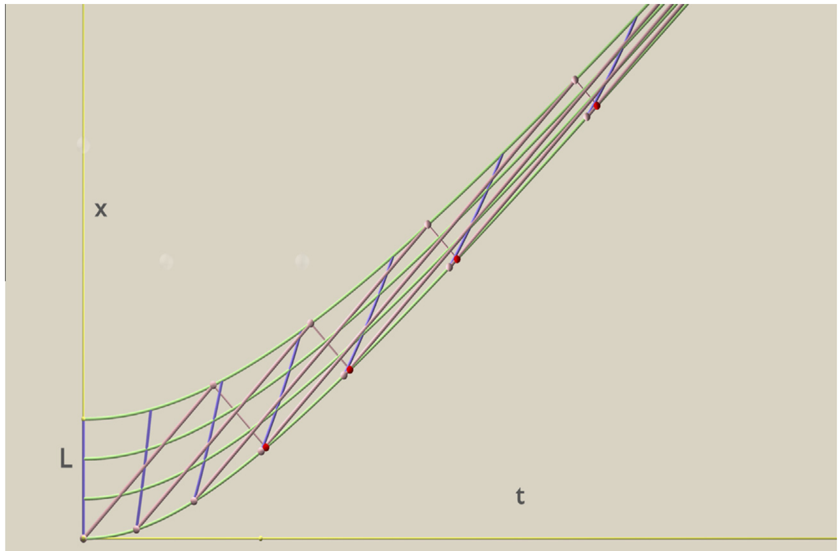

Fig. 1. Home frame world-surface of a ‘rigor mortis’ acceleration medium with diagonal radar trajectories and tilted curved fixed velocity loci.

$$\frac{e^{-\check{\rho}\alpha_f}}{\alpha_f}-\frac{1}{\alpha_f}+\frac{1}{\alpha_r}+L=\frac{e^{-\grave{\rho}\alpha_r}}{\alpha_r}. \tag{8}$$

Multiplying (6) and (7)i we obtain THE FORWARD GENERAL FIXED THRUST ROCKETS’ RADAR EQUATION:

$$\frac{1}{\alpha_f^2}=\left[\frac{e^{-\acute{\rho}\alpha_r}}{\alpha_r}+\frac{1}{\alpha_f}-\frac{1}{\alpha_r}-L\right]\left[\frac{e^{\grave{\rho}\alpha_r}}{\alpha_r}+\frac{1}{\alpha_f}-\frac{1}{\alpha_r}-L\right]. \tag{9}$$

Multiplying (7)ii and (8) we obtain THE REVERSE GENERAL FIXED THRUST ROCKETS’ RADAR EQUATION:

$$\left[\frac{e^{\hat{\rho}\alpha_f}}{\alpha_f}-\frac{1}{\alpha_f}+\frac{1}{\alpha_r}+L\right]\left[\frac{e^{-\check{\rho}\alpha_f}}{\alpha_f}-\frac{1}{\alpha_f}+\frac{1}{\alpha_r}+L\right]=\frac{1}{\alpha_r^2}. \tag{10}$$

The ‘rigor mortis’ accelerating medium

Radar intervals and space/time dispersals

Applying the special own-accelerations condition $L=\frac{1}{\alpha_f}-\frac{1}{\alpha_r}$ i.e. $\alpha_f=\frac{\alpha_r}{1+L\alpha_r}$ and $\alpha_r=\frac{\alpha_f}{1-L\alpha_f}$, results in *constant radar interval and zero time dispersal values* (⇒ Section “ ‘Rigor mortis’ acceleration ” above). From forward radar Eq. (9) $e^{(\grave{\rho}-\acute{\rho})\alpha_r}=\frac{\alpha_r{}^2}{\alpha_f^2}=(1+L\alpha_r)^2$ i.e.

THE FORWARD ‘RIGOR MORTIS’ RADAR INTERVAL

$$\grave{\rho}-\acute{\rho}=\frac{2}{\alpha_r}\ln(1+L\alpha_r)=\frac{2}{\alpha_r}\ln\left(\frac{\alpha_r}{\alpha_f}\right). \tag{11}$$

Also from reverse radar Eq. (10) $e^{-(\check{\rho}-\hat{\rho})\alpha_f}=\frac{\alpha_f{}^2}{\alpha_r^2}=(1-L\alpha_f)^2$ a second constant value emerges—THE REVERSE ‘RIGOR MORTIS’ RADAR INTERVAL:

$$\check{\rho}-\hat{\rho}=-\frac{2}{\alpha_f}\ln(1-L\alpha_f)=\frac{2}{\alpha_f}\ln\left(\frac{\alpha_r}{\alpha_f}\right). \tag{12}$$

Moreover $$\frac{\grave{\rho}-\acute{\rho}}{\check{\rho}-\hat{\rho}}=\frac{\alpha_f}{\alpha_r}=\frac{1}{1+L\alpha_r}=1-L\alpha_f. \tag{13}$$

Remark I. THE ‘RIGOR MORTIS’ FORWARD AND REVERSE RADAR INTERVALS ARE CONSTANT AND THEIR RATIO EQUALS THE ROCKET ACCELERATIONS’ RATIO α_f/α_r.

For the rockets at any set velocity v, from (1) their home frame *time dispersal* is:

$$t_f-t_r=\gamma v\left[\frac{1}{\alpha_f}-\frac{1}{\alpha_r}\right]=\gamma vL. \tag{14}$$

From (4) the corresponding *distance dispersal* is:

$$(x_f+L)-x_r=\frac{\gamma-1}{\alpha_f}+L-\frac{\gamma-1}{\alpha_r}=\gamma\left[\frac{1}{\alpha_f}-\frac{1}{\alpha_r}\right]-L+L=\gamma L. \tag{15}$$

The Lorentz transformations and Eqs. (14) and (15) thus yield the corresponding comoving frame time dispersal

$$\gamma\left[(t_f-t_r)-v(x_f+L-x_r)\right]=\gamma[\gamma vL-v\gamma L]=0 \tag{16}$$

and distance dispersal

$$\gamma\left[(x_f+L-x_r)-v(t_f-t_r)\right]=\gamma\left[\gamma L-v^2\gamma L\right]=L. \tag{17}$$

The ‘rigor mortis’ home frame world-surface

Denoting the rear rocket as ‘increment l_0', intermediate medium increments can be identified by their relative launch length l from the rear rocket l_0 $(0\leqslant l\leqslant L)$. For convenience we may set $\alpha_r=1$ (whereby time and lengths are *rescaled* so that α_r as well as c are *one*) and consider each arbitrary medium increment l accelerating at $\alpha=1/(1+l)$ with each increment’s curve elevated from start at $x_0=l$.

Eq. (5) then yields $((x-l)\alpha+1)^2-(t\alpha)^2=1$ i.e. $(x-l+1+l)^2-t^2=(1+l)^2$.

THE HOME FRAME’S RIGOR MORTIS MEDIUM’S WORLD-SURFACE EQUATION

$$(x+1)^2-t^2=(1+l)^2. \tag{18}$$

Fig. 1 shows the computer generated home frame ‘rigor mortis’ world-lines for the rear and front rockets and intermediate *likewise differently accelerating* medium increments. The further the distance from the rear rocket, the slower the increment’s acceleration. Emitted and reflected radar trajectories appear as diagonal lines (scaled $c=\pm1$). Substituting $L=0.57$, $a_r=1$ and $a_f=a_r/(1+L.a_r)=0.637$ in Eq. (11) and dividing by the chosen rocket own-time period $3\pi/32$ between respective emissions yields 3.063. This clearly corresponds to each of the chart’s radar response intervals in terms of the rear rocket’s own-time emission interval.

Fixed velocity 'rigor mortis' loci

The *tilted* fixed velocity loci traced at regular rocket clock own-time intervals, simply connect each individual increment's hyperbola point corresponding to the respective shared home frame velocity. Merely 'schematically' represented by *straight rectangular strips* in textbook [13]'s Fig. 3.3 (p. 72), such loci are (apart from at launch) *curved*. Thus for each rear rocket own-time $\tau_n = n\Delta\tau$, the coordinates for increment l_i (whereby $\alpha_i = 1/(1+l_i)$) are[4]:

$$t_i = \sinh\left(\frac{\tau_n}{1+l_i}\right)\cdot(1+l_i) \qquad \text{and} \qquad x_i = \left[\cosh\left(\frac{\tau_n}{1+l_i}\right)-1\right](1+l_i)+l_i. \tag{19}$$

These represent co-moving frame's increments sharing identical velocities (Eq. (16)), whose distributed lengths add up—*simultaneously in the particular comoving frame*—to the constant unchanging launch separation L (Eq. (17)).[5]

The differential inverse own-acceleration condition

The Minkowski metric, which reduces in the present context to $\frac{\Delta\chi}{\Delta\tau} = 1$, does actually apply here because, as Eqs. (16) and (17) show, the comoving frame is shared all along the medium's length.

Remark II. TWO OBSERVERS OF EVERY COINCIDENTALLY COMOVING FRAME COULD RESPECTIVELY AND SIMULTANEOUSLY OBSERVE ACCELERATING ROCKETS IN 'RIGOR MORTIS' MODE I.E. MOMENTARILY COINCIDENT AND STATIONARY AND WITH SEPARATION AND INTER-ROCKET RADAR INTERVALS REMAINING UNCHANGED, IF AND ONLY IF THE DIFFERENTIAL INVERSE OWN-ACCELERATIONS CONDITION APPLIES.

Note: If the own-accelerations condition were *not* met, then the equations *would not reflect* Fig. 1's home frame world-surface *and vice-versa*. Likewise, as will be further elaborated upon in Sections "Homogeneous acceleration" and "The misapplied 'edict' ", the Minkowski metric cannot be valid *except* under these specific 'rigor mortis' conditions or for point objects.

Homogeneous acceleration

Inter-rocket radar intervals

If $\alpha_f = \alpha_r = \alpha = 1$ (whereby time and lengths are *rescaled* so that c as well as α (now everywhere the same) are *one* i.e. for truly uniform acceleration, forward and reverse radar Eqs. (9) and (10) respectively reduce to:

$$e^{\grave\rho} = \frac{1}{[e^{-\acute\rho}-L]}+L \qquad \text{and} \tag{20}$$

$$e^{\check\rho} = 1\Big/\left[\frac{1}{[e^{\hat\rho}+L]}-L\right] = \left[\frac{[e^{\hat\rho}+L]}{1-L(L+e^{\hat\rho})}\right]. \tag{21}$$

So for $\acute\rho < \ln(1/L)$, THE UNIT ACCELERATION REAR ROCKET'S RADAR INTERVAL

$$\grave\rho - \acute\rho = \ln\left[\frac{1}{e^{-\acute\rho}-L}+L\right]-\acute\rho. \tag{22}$$

Likewise (from (21)) assuming that for rear rocket re-emission time $\grave\rho, e^{-\grave\rho} > L$:

THE UNIT ACCELERATION FRONT ROCKET'S RADAR INTERVAL[6]

$$\check\rho - \hat\rho = \ln\left[\frac{1+Le^{-\hat\rho}}{1-L(L+e^{\hat\rho})}\right]. \tag{23}$$

Remark III. As not generally appreciated, 'radar distance' between identically accelerating rockets varies i.e. THE SECOND POSTULATE DOES NOT APPLY FOR EXTENDED OBJECTS UNDER HOMOGENEOUS ACCELERATION (⇒ Sections "The misapplied 'edict' " and "Conclusion").

The homogeneously accelerating medium's world-surface

We imagine an idealized medium between the rockets with each part accelerating with the same identical unit thrust as the two rockets themselves (thus not involving any forces or delays). An approximation of such a medium has been described by *Podosenov* [16] as *"an equilibrium of charged dust in parallel electric and gravitational fields"* (⇒ Section "The misapplied 'edict' "). Eq. (5) gives us

THE HOME FRAME HOMOGENEOUS WORLD-SURFACE EQUATION

$$(x-l+1)^2 - t^2 = 1. \tag{24}$$

Each increment's *travelled* x of Eq. (24) is represented on the home frame world surface by the corresponding *position* $x+l$ value which, replacing x in (24) and using (1) and (4), yields identity $\cosh^2\tau - \sinh^2\tau \equiv 1$. The world-surface and corresponding hyperbolic world-lines are shown in Fig. 2, with trajectories of photons emitted from the rear rocket and reflected back from the identically accelerating front rocket.

The vertical lines represent the medium itself at equal *rocket own-time* intervals $\Delta\tau$. Substitution of $L = 0.5548$, $\Delta\tau = 3\pi/32$, $\acute\rho_0 = 0$ and $\acute\rho_1 = 3\pi/32$ in radar Eq. (22) yields: $\grave\rho_0 - \acute\rho_0 = \frac{3\pi}{32}\cdot 3.497$ and $\grave\rho_1 - \acute\rho_1 = \frac{3\pi}{32}\cdot 4.977$. These intervals correspond to those in the computer generated diagram where emitted and reflected photon trajectories are *straightline* $\pm 45^\circ$ *diagonals* (just as in Fig. 1). The fixed velocity loci in this case are just straight lines.

The 'asymptotic horizon'

An important matter traditionally treated only in general relativity literature is 'the fate' of photons emitted from the rear rocket. In this context, we define

AN EMITTED PHOTON'S ASYMPTOTIC HORIZON

$$L_h \triangleq e^{-\hat\rho}. \tag{25}$$

As $\hat\rho$ is *the photon's front rocket arrival time*, for $\alpha_r = \alpha = 1$ Eq. (6) then yields

THE REFLECTION PHOTON EQUATION

$$e^{\check\rho} = \frac{1}{e^{-\hat\rho}-L} = \frac{1}{L_h - L}. \tag{26}$$

For a photon emitted at *asymptotic horizon emission home time* $\underline{t} = \sinh\underline{\rho} = (1/L - L)/2$ i.e. $e^{-\underline{\rho}} = L_h = L$, $e^{\check\rho}$ would be infinite. The photon asymptotically approaches but never actually reaches the front rocket and so is not reflected. Neither of course would a photon emitted after that time instant be reflected. Note that in Fig. 2, the third outgoing trajectory (in black) is that of a 'horizon photon' where $\acute\rho_2 = 2\cdot\frac{3\pi}{32} = 0.5890$, the asymptotic horizon

[4] The final l_i must be added since each increment hyperbola starts at $x = l_i$.

[5] Of course in the home frame the rockets observed at any set v value are separated by L/γ, but such observations are non-simultaneous in that frame (⇒ Section " The front/rear rockets' 'time dispersals' ").

[6] We recall that the photon equations preceding Eq. (6) for example, are concerned with *one fixed home frame* whereas the homogeneous acceleration variable intervals (22) and (23) cover *multiple changing comoving frames*

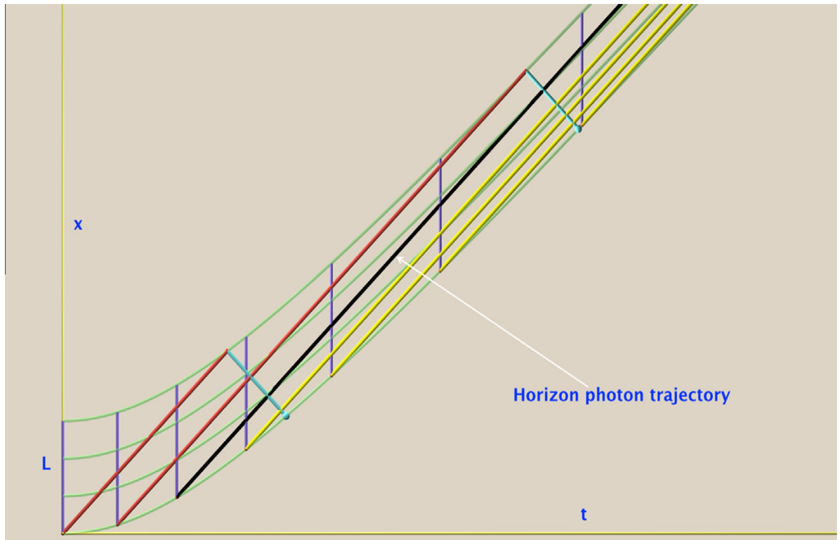

Fig. 2. Home frame world-surface of a homogeneously accelerating medium, with reflected and nonreflected radar trajectories and fixed velocity loci.

distance $L_h = e^{-\acute{\rho}_2} = 0.5548$ equals the chosen launch separation distance L. Hence only the first two photons ($\acute{\rho}_0 = 0$ and $\acute{\rho}_1 = \frac{3\pi}{32} = 0.2945$) are actually reflected.

Remark IV. A photon emitted at launch from a rear rocket at scaled unit rest length ($\acute{\rho} = 0, L = L_h = e^0 = 1$) from the front rocket, would never actually reach the front rocket which would be approached asymptotically as $\tau \to \infty$. With an earth gravity thrust, such a length would be $c^2/10 \approx 9 \cdot 10^{15}$ meters—coincidentally just under one light year. Hence A ROCKET PERMANENTLY ACCELERATING FASTER THAN 1g NEVER SEES AN EVENT OCCURRING MORE THAN A LIGHT YEAR AWAY IN ANY OF THE ROCKET'S COMOVING FRAMES.

As *Marolf* wrote in 2003 [10]: "*merely by undergoing uniform acceleration, the rocket ship has cut itself off from communication with a large part of the spacetime.*".

Time dispersals and retrospective distances

A rockets shutdown scenario

Let us imagine that unit acceleration rear rocket r and front rocket f shut off their motors simultaneously in home frame $\underline{H}$ as they travel at fixed speed $\breve{v} = \tanh \breve{\tau}$ in that frame—the overhead *breve* denoting *shutdown*. They each 'dock' at respective spacestations R and F which both happen to be moving at the same home frame $\breve{v}$ speed. This is illustrated in Fig. 3, showing the nonaccelerating spacestations' straight world-lines tangential to the arriving rockets' world-line hyperbolae sections and subsequently overlapping the then inertial rockets' straight world-line sections.[7]

The symmetrical spacetime chart viewpoint

We now view this scenario in a *symmetrical dual frames chart* equivalent to the Loedel/Brehme charts familiar in the literature (e.g. Loedel [3], Brehme [4], Sartori [7]). Fig. 4 shows the inertial launch 'home' frame $\underline{H}$'s light red reference $x|t$ axes and final comoving 'shutdown' frame $\underline{\breve{C}}$'s light blue reference $X|T$ axes. Rear rocket launch event $r0$ (our chart's 'reference origin') and front rocket launch event $f0$[8] are simultaneous in frame $\underline{H}$. Also shown are *straight* world-lines (transparent white) of the two inertial spacestations R and F each at related shutdown speed $\breve{v}$ relative to the home frame and thus stationary in the rockets' shutdown comoving frame $\underline{\breve{C}}$ (their world-lines being parallel to the chart's T-axis). Both rockets progress through continually varying intermediate comoving frames at constant unit own-acceleration, as represented by the chart's *curved* yellow world-lines.

We mentioned in Section "Spacetime parameters of a fixed thrust rocket" an accelerating rocket's 'retrodistance' χ *from its launch position* $x_0 = 0$, measurable simultaneously in its comoving frame i.e. with $\Delta\tau = 0$. Applying *the inverse Lorentz transformation* $x = (\chi + \tau v)\gamma$, yields $\chi = x/\gamma$. This is exemplified for the rear rocket's arrival event rR in Fig. 4 (previously launched with its x, χ, t and τ parameters all zero), whereby $\chi_{rR} = x_{rR}/\gamma$. Hence using Eq. (4):

ROCKET FRAME PERCEIVED 'RETRODISTANCE'

$$\chi = 1 - 1/\cosh\tau = 1 - 1/\gamma. \quad (27)$$

Acceleration phase comoving frame $\underline{C}$ variables $\chi = 1 - 1/\cosh\tau$ and $\tau = \sinh^{-1} t$, are not directly represented in the chart—in contrast to home frame $\underline{H}$ variables $x|t$ and shutdown frame $\underline{\breve{C}}$ variables $X|T$.[9] Incidentally, it is worth noting that the *acceleration phase* congruent curved world-line segments connecting events $r0$ and rR and events $f0$ and fF respectively, are horizontally *symmetrical*. These curves are 'mappings' of the rockets' home frame *asymmetric* hyperbolae of Fig. 2—geometrically transformed as a dual inertial frame chart's *symmetric* world-lines.

As perceived in shutdown comoving frame $\underline{\breve{C}}$, at launch rear rocket r is ahead of nonaccelerating spacestation R and moves *backward* as its speed in $\underline{\breve{C}}$ tends toward zero. Thus, just as its motor is shut down, r 'backs onto' R (event rR) and remains 'docked'—their two world-lines *overlap from then on*.[10] Likewise front rocket f 'backs onto' (event fF) the other inertial spacestation F also

[7] This scenario was first outlined in an interesting 1968 book co-authored by *Brehme* [4] (Fig. 5, p. 80). More recently, it was also considered by *Redžić* [15].

[8] Spacetime frames we denote by underlined capitals, and events by double characters e.g. fF means rocket f's arrival at spacestation F.

[9] The acceleration phase world-lines' vertical (Z) and horizontal (Y) coordinates of Fig. 4, relate as: $Y = t\cos\frac{\breve{\phi}}{2} - x\sin\frac{\breve{\phi}}{2}$ and $Z = x\cos\frac{\breve{\phi}}{2} - t\sin\frac{\breve{\phi}}{2}$, where the inter-axes shutdown velocity angle $\breve{\phi} = \arcsin\breve{v}$.

[10] Were the rockets to have continued accelerating, then after arrival each would have proceeded in a *forward* direction away from its spacestation.

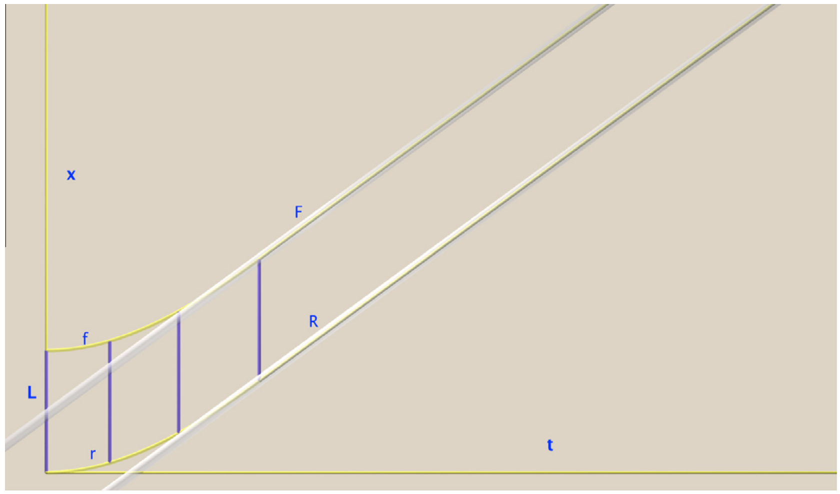

Fig. 3. Home frame hyperbolic world-lines of uniformly accelerating rockets docking at two spacestations.

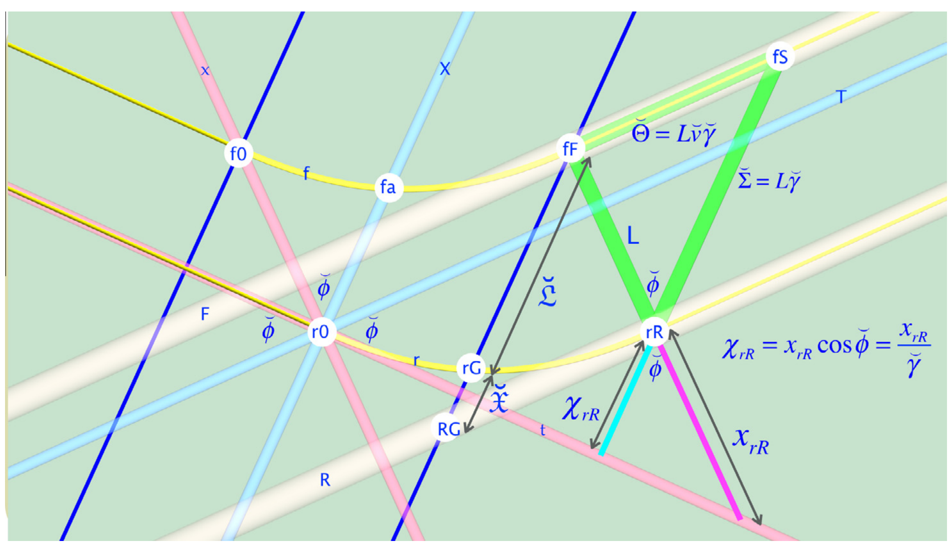

Fig. 4. Loedel/Brehme chart equivalent with rocket and spacestation world-lines and shutdown frame space and time 'dispersals triangle'.

moving at home-frame related speed $\breve{v}$, and likewise remains docked.

The front/rear rockets' 'time dispersals'

A crucial—nonintuitive—spatio-temporal issue in an extended medium's acceleration, is inadequately appreciated *'time dispersal'*. In accordance with the Lorentz transformation, identically accelerating rockets launched together a 'launch distance' L apart, are 'imagined'[11] to increasingly disperse a distance $L\gamma = L\cosh\tau$ apart in each respective ever changing comoving inertial frame. Yet and likewise by virtue of the Lorentz transformation, their clocks—*as could be simultaneously related by comoving frame observers*—become increasingly *temporally disjoint* by factor $Lv\gamma = L\sinh\tau$. Shutdown events *rR* and *fF* are therefore 'time dispersed' by $\breve{\Theta} \triangleq L\breve{v}\breve{\gamma} = L\sinh\breve{\tau}$ in the 'shutdown frame' $\breve{C}$. From the viewpoint of the front rocket now stationary in frame $\breve{C}$, the rear rocket, which suddenly 'acquires' a different comoving *'shutdown gap' frame* $\breve{G}$—an event we label *rG*, still continues to move alone *backward* at a continually decreasing speed. This continues until the ultimately 're-synchronized' rockets are *both* stationary in the shutdown frame and separated by 'dispersal distance' $\breve{\Sigma} \triangleq L\breve{\gamma}$.

Fig. 4's 'DISPERSALS TRIANGLE' $(rR - fF - fS)$ represents the rockets' launch separation L, *initial* shutdown time dispersal $\breve{\Theta} = L\breve{v}\breve{\gamma} = L\tan\breve{\phi}$ and *ultimate* synchronization spatial separation $\breve{\Sigma} = L\breve{\gamma} = L/\cos\breve{\phi}$. We call event *fS* the front rocket's 'RE-SYNCHRONIZATION' event. At event *fS*, spacestation *F*'s docked front rocket's own-time will be $\tau_{fS} = \tau_{fF} + \breve{\Theta} = \breve{\tau} + L\sinh\breve{\tau}$.

The front rocket's 'retrospective separation'

In the mentioned INTERMEDIATE SHUTDOWN INERTIAL 'GAP' FRAME $\breve{G}$, rear rocket's clock initially (at event *rG*) reads not $\breve{\tau}$ but $\breve{\tau} - L\sinh\breve{\tau}$ *due to time dispersal*. Imagining it having a second clock $\mathfrak{T}$ *zeroed* at event *rG*, the distance $\breve{\mathfrak{X}}$ which the unit thrust rear rocket has yet to travel in frame $\breve{C}$ until it becomes momentarily stationary therein (as its primary clock reads $\breve{\tau}$ and its secondary clock reads $\breve{\mathfrak{T}} = L\sinh\breve{\tau}$), can be obtained using Eq. (27). This distance we call

[11] i.e. by virtue of home frame (mis)interpreted 'inverse contraction'.

THE REAR ROCKET'S DISPERSAL TIME SHUTDOWN GAP DISTANCE:

$$\breve{\mathfrak{X}} = 1 - \frac{1}{\cosh \breve{\mathfrak{T}}} = 1 - \frac{1}{\cosh (L \sinh \breve{\tau})}. \qquad (28)$$

Subtracting $\breve{\mathfrak{X}}$ from frame $\breve{\underline{C}}$'s spatial dispersal $L\breve{\gamma}$ yields (for *any* front rocket own-time τ): THE FRONT ROCKET'S RETROSPECTIVE INTER-ROCKET SEPARATION:

$$\mathfrak{L} = L\gamma - \mathfrak{X} = L \cosh \tau + \frac{1}{\cosh (L \sinh \tau)} - 1. \qquad (29)$$

Significantly, whereas dispersal time shutdown gap distance $\mathfrak{X}$ tends toward the maximum asymptotic horizon unit length limit as $\tau \to \infty$ ($\Rightarrow$ Section "The 'asymptotic horizon'"), retrospective separation $\mathfrak{L}$ tends toward infinity—i.e. beyond the unit scaled length limit.

One might think this solves the question of length in accelerating frames. Of course if things were that easy, the matter would have been long resolved. However, in the shutdown comoving frame $\breve{\underline{C}}$, from the momentarily stationary front rocket's viewpoint the rear rocket *still continues to move backward and its clock is out of synchronism*.

Remark V. THE INERTIAL LENGTH MEASUREMENT CRITERION OF A MEDIUM NEEDING TO BE 'AT REST' IN AN INERTIAL FRAME IS INFRINGED. EXCEPT IN THE 'RIGOR MORTIS' CASE WHEN $\Lambda = \mathfrak{L} = L$, COMOVING FRAME 'UNILATERAL SEPARATION' $\mathfrak{L}$ AND 'NONINERTIAL OWN-LENGTH' Λ (YET TO BE ASCERTAINED) ARE NOT EQUIVALENT ($\Rightarrow$ [24]). Notably, as long as both rockets continue to accelerate i.e. prior to any shutdown, the situation is markedly different (as already mentioned for example by Redžić [15]). The 'dispersal time' to 'dispersal distance' ratio equals scaled velocity v which approaches limit *one* as home frame time $t \to \infty$.

Photon crossing rates

Let us consider a simple *Gedankenexperiment*. Increments of the homogeneously accelerating extended idealized medium are somehow outfitted with clocks whose readings could be recorded by a third party as they are individually crossed by a photon (itself 'timeless'). A photon's third party observer would consecutively record ever increasing readings of increment clocks whose comoving frame continually *changes*.

We denote an increment's 'noninertial own-length' from the medium's rear end as $\lambda(l, \tau) = l \cdot \epsilon(\tau)$ $(0 \leqslant l \leqslant L)$, where $\epsilon(\tau)$ (whatever it might be) is the homogeneous *expansion factor*. The whole medium's 'noninertial own-length' will be $\lambda(L, \tau) = L \cdot \epsilon(\tau)$. Now we define 'THE MEDIUM-TIMED PHOTON CROSSING RATE' as *the rate of change of this increment's length* $\lambda(l, \tau)$*, as viewed from the timeless photon's third party's perspective of crossed increment own-times* i.e. $\partial\lambda/\partial\tau$. The crossing rate should NOT of course be confused with the unit limit speed of the photon itself as perceived in an increment's inertial momentary comoving frame. For an *outgoing* photon arriving at an *arbitrary* increment *l*, replacing $\hat{\rho}$ by τ and L by l in Eq. (6) (with $\alpha_f = \alpha_r = 1$) gives $e^{-\tau} = e^{-\hat{\rho}} - l$. Partial differentiation yields $\partial l/\partial\tau = e^{-\tau}$. Also $\partial\lambda/\partial l = \epsilon(\tau)$. Hence

AN EMITTED PHOTON'S MEDIUM-TIMED CROSSING RATE

$$\frac{\partial\lambda}{\partial\tau} = \frac{\partial\lambda}{\partial l} \cdot \frac{\partial l}{\partial\tau} = \epsilon(\tau) \cdot e^{-\tau}. \qquad (30)$$

A photon will tend to be ultimately 'surfed'[12] by an accelerating medium's increment approaching unit limit speed in the inertial home frame. Hence, whatever the co-accelerating medium's expansion factor $\epsilon(\tau)$ might be, as the rockets' own-time τ tends ('in unison') toward infinity, by virtue of Eq. (30) the outgoing photon crossing rate will tend toward zero.

THE MEDIUM-TIMED FORWARD PHOTON CROSSING RATE IN THE LIMIT

$$\left.\frac{\partial\lambda}{\partial\tau}\right|_{\tau\to\infty} = \epsilon(\tau) \cdot e^{-\tau}|_{\tau\to\infty} = 0. \qquad (31)$$

If reflected, the photon travels *backward* to meet an arbitrary increment l at home time $t = \sinh\rho$ and home *position* $x + l = \cosh\tau - 1 + l$, over equal home-frame time and distance intervals $t - \hat{t} = (\hat{x} + L) - (x + l)$. Substituting, $\sinh\tau - \sinh\hat{\rho} = (\cosh\hat{\rho} - 1 + L) - (\cosh\tau - 1 + l)$ i.e. $L - l = e^{\tau} - e^{\hat{\rho}}$ so $\partial l/\partial\tau = -e^{\tau}$.

A REFLECTED PHOTON'S MEDIUM CROSSING RATE

$$-\frac{\partial\lambda}{\partial\tau} = -\frac{\partial\lambda}{\partial l} \cdot \frac{\partial l}{\partial\tau} = \epsilon(\tau) \cdot e^{\tau}. \qquad (32)$$

Finally we take into account the ultimate tendency of a *reflected* photon to backward traverse the entire forward moving medium *momentarily* i.e. 'cross' it at a speed *approaching infinity*. THE BACKWARD PHOTON'S LIMIT CROSSING RATE

$$-\left.\frac{\partial\lambda}{\partial\tau}\right|_{\tau\to\infty} = \epsilon(\tau) \cdot e^{\tau}|_{\tau\to\infty} = \infty. \qquad (33)$$

Remark VI. Photon's medium-timed crossing rates *are always less than one and decreasing toward zero* for *co-directional* photons, and *greater than one and increasing toward infinity* for *counter-directional* photons.

Remark VII. Prior to rocket motor shutdowns, expansion $\epsilon(\tau)$ is not the 'gamma' factor—contrary to what is widely held—since as $\tau \to \infty, \gamma \cdot e^{-\tau} = \cosh\tau \cdot e^{-\tau} = 0.5$.

Remark VIII. Whatever the co-accelerating medium's expansion factor $\epsilon(\tau)$ might be, as the rockets' own-time τ tends ('in unison') toward infinity ($\Rightarrow$ [20]), it must conform to Eqs. (31) and (33):

A HOMOGENEOUSLY ACCELERATING MEDIUM'S EXPANSION CONDITIONS

$$\text{as} \quad \tau \to \infty, \quad \epsilon(\tau) \cdot e^{-\tau} = 0 \quad \text{and} \quad \epsilon(\tau) \cdot e^{\tau} = \infty. \qquad (34)$$

The misapplied 'edict'

The above considerations challenge the astonishing prevalence of the Minkowski metric—an 'exquisite test' in the recent words of a mainstream journal reviewer's unabashed judgement—not only as the often asserted basis of general relativity, but also with respect to special relativity. As a paramount example, in the classic *The Geometry of Minkowski Spacetime* [19] (1992, 2010, pp. 2–4), *Naber* discusses separate 'admissible' observers' spacetime frames wherein *"photons propagate rectilinearly with [scaled] speed 1"*, yet appears to nowhere explicitly address the issue of *a homogeneously accelerating extended medium* either in a flat spacetime or in a curved spacetime context.

Where 'rigor mortis' acceleration is concerned, observers of each comoving frame experience zero relative velocities and identical home frame times (16) along the medium, as well as unchanging length (17) and radar intervals ((11) and (12)). This implies that a photon's medium timed crossing rate will always be the limit speed, and—accordingly—the Minkowski metric IS satisfied. Conversely and notwithstanding the latter's widely unquestioned adoption by the physics community, OTHERWISE IN GENERAL NEITHER CRITERION APPLIES. This is *especially* clearly the case where a medium subject to *homogeneous* i.e. uniform acceleration is concerned.

[12] The third party observer would report that the photon took ever longer increment clock intervals to cross the medium i.e. $\partial\lambda/\partial\tau$ would tend ultimately to approach the value zero.

Podosenov's description (⇒ Section "The homogeneously accelerating medium's world-surface" above) of an idealized homogeneously accelerating medium arguably *alone* illustrates why a 'rigor mortis' accelerating medium conforming to Minkowski's metric is inappropriate for analyzing Einstein's equivalence principle—by virtue of the *inherent nonhomogeneity* of the latter medium.

As well as challenges by *Brown and Pooley* in *Minkowski spacetime: a glorious non-entity* [11] (2004) and (as mentioned above) by *Callahan* ([8]), a recent paper [21] by five authors discussing the metric's relevance to Einstein's general relativity field equations, mentions that only a *second order* of exactness is achieved i.e. *the correlation is not in itself even analytic.*

Conclusion

The 'Rindler coordinates' treatment, which to all appearances deploys Minkowki's metric indiscriminately (⇒ https://en.wikipedia.org/wiki/Rindler_coordinates), was not mentioned in [18] nor even alluded to in an often cited 1987 mainstream journal paper *Uniformly accelerated reference frames in special relativity* [6]. Ironically the latter earnest and much cited paper itself had a significant flaw. Adopting the widespread *misinterpretation* of the second postulate whereby a photon has constant speed in all inertial frames (the present paper's Eqs. (10) and (11) show that radar intervals differ for *noninertial* frames), the authors wrongly assumed that the radar interval i.e. *"the elapsed time* $\tau_3 - \tau_1$ *for the round trip...should be constant"* ([6] p.255, sections IV and V). This led them to incorrectly infer that *"the proper [i.e. own-] distance between any two [identically accelerating] observers remains constant."*

Nevertheless Florida physicists *Desloge* and *Philpott's* severe 'voice in the wilderness' censure in 1987 of the status quo ([6] Section I) deserves being recalled. Unfortunately to no avail, they aptly identified endemic weaknesses of the opaque paths generally employed in relativity literature dealing with accelerating frames:

> *"The approach is unnecessarily formal or abstract, key concepts are left undefined, a working knowledge of general relativity is assumed, no attempt is made to give a physical interpretation of the coordinates introduced, the relationship between different sets of coordinates used is not made, and no investigation of the properties of the frame is made."*

The present paper has been restricted to establishing an accelerating medium's bi-directional radar formulae, with unequivocal conclusions drawn regarding such a medium's 'noninertial length' expansion (⇒ [20,24]). These indicate that the Minkowski metric—the starting 'edict' of many general relativity textbooks—should not be adopted as an absolute principle in either special or general relativity theory.

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